Developing teaching of mathematics to first year engineering students

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Engineering Students Understanding Mathematics (ESUM) is a developmental research project at a UK university. The motivating aim is that engineering students should develop a more conceptual understanding of mathematics through their participation in an innovation in teaching. A small research team has both studied and contributed to innovation, which included small group activity, a variety of forms of questioning, an assessed group project and use of the GeoGebra medium for exploring functions. Perspectives of community of practice and inquiry, and documentational genesis underpin the research approach. Issues and findings to date from the project are presented.

1. Background to ESUM—Engineering Students Understanding Mathematics

A mathematics module for Materials Engineering students in a UK university has run for 3 years with the same lecturer and modifications to its presentation each year. These modifications have intended to create a more student-participative module and encourage students to develop more conceptually based understandings of mathematics. Modifications in previous years have had limited success (Jaworski, 2008, 2010) and the innovation this year has been designed to be more coherent and far-reaching, encompassing changes to how the module is delivered and the ways in which students interact with the mathematics, the lecturer and each other. Innovation has been undertaken by a research team of three teachers of mathematics (two with extensive experience of teaching engineering students and one, the lecturer, with extensive experience of mathematics teaching and teacher education at secondary level) who design, conduct and reflect on the teaching (the insiders—Bassey, 1995), and a research officer (outsider) who has collected and analysed data as agreed with the teaching colleagues.

The module has been taught by one of the team (the lecturer) over 13 weeks with two lectures and one tutorial per week. This year’s cohort was 48 students, most of whom had A level mathematics with grades A–C, just a few with alternative qualifications and two with no mathematics since GCSE. Lectures were timetabled in tiered lecture theatres. The weekly tutorial was held in a large computer laboratory with individual computer tables in squares of four, each set of tables accommodating one group of students.

For the tutorials, students were grouped in threes and fours and expected to work together on set tasks and an assessed project. Tasks and projects were designed specially for the module by the
teaching team and formed a part of the innovation. Both included inquiry-based questions designed to encourage exploration in mathematics using GeoGebra. In addition, inquiry-based questions were used in lectures along with more traditional questions to encourage student involvement and provide feedback on understanding. Question design drew on a range of published resources.

2. Developmental research in ESUM

Research was designed both to promote development and to study it (Jaworski, 2003). Promotion was achieved by feeding back to teaching as data were collected and by creating an inquiry approach to teaching. The research studied the entire process through a rigorous analysis of data collected. The project had four phases. A design phase (of questions and tasks) preceded teaching and continued in parallel with the teaching phase for 13 weeks. Two PhD students contributed to location and design of questions and tasks. Practitioner reflection, data collection and a first level of analysis coincided with these two phases. The lecturer was a practitioner-researcher, reflecting on all activity and feeding back from observations and other data to ongoing teaching design and practice. The outsider researcher observed lectures and tutorials, with audio-recordings of lectures. She designed and administered two questionnaires for student data and feedback from teaching sessions and, with another member of the team, has held one–one and focus group interviews with students at the end of the teaching semester. All research instruments and activity were agreed first with the teaching team. At the end of the teaching semester, a phase of data analysis has taken place involving mainly the researcher and one member of the teaching team. A final phase will involve dissemination of findings and their use in the (re)design of the module for the forthcoming year.

3. Theoretical perspectives underpinning project design

The ESUM project has drawn on theory of communities of inquiry developed in research into teaching development at school levels (e.g. Jaworski, 2006). Research took place within an institutional environment, the university, with its norms and expectations which can be seen to form an established community of practice (Wenger, 1998). This includes curricula and assessment, lectures and tutorials, attitudes and approaches to teaching, students’ perspectives and the physical constraints of teaching spaces and timetables. Inquiry was introduced at several levels including inquiry-based mathematical tasks, inquiry into teaching approaches and design of questions, and research inquiry as conducted by the research team as a whole. Since the team belonged to the community of practice, it engaged with practice, used imagination in interpreting practice and aligned with the norms of practice (Wenger, 1998). In introducing inquiry at the various levels, it engaged with critical alignment in which necessary alignment within the community of practice was questioned to effect change through innovation (Jaworski, 2006).

The teaching team engaged deeply in design of teaching which included key resources (such as inquiry-based questions and GeoGebra), forms of pedagogy (such as small group activity and assessed project work), reflection and feedback, and critical analysis of ongoing practice and learning through research. Theory of Documentational Genesis (Gueudet & Trouche, 2011) proved valuable in conceptualizing the activity of innovation to promote more conceptual student understanding. Here, a document derives from a set of resources together with a scheme of utilization. Genesis means becoming: becoming a mathematics teacher; becoming a professional user of resources; becoming a knowledgeable professional. In his book Communities of Practice, Wenger talks of learning as ‘a

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1GeoGebra: http://www.geogebra.org/cms/
process of becoming’ (1998, p. 215). This, he claims, is ‘an experience of identity’ (1998, p. 215), where identity ‘serves as a pivot between the social and the individual, so that each can be talked about in terms of the other’ (1998, p. 145). Wenger suggests that ‘learning as participation . . . takes place through our engagement in actions and interactions’ and ‘embeds this engagement in culture and history’ (1998, p. 13). Documentational genesis, a term which captures the process of the mathematics teacher becoming a professional user of resources and, concomitantly, a knowledgeable professional, navigates the ground between the personhood of the teacher and the teacher’s belonging (Wenger, 1998) to social structures and communities in which resources take meaning. Visnovska et al. (2011) write:

[T]eachers’ documentation work includes looking for resources (e.g., instructional materials, tools, but also time for planning, colleagues with whom to discuss instructional issues, and workshops dedicated to specific themes) and making sense and use of them (e.g., planning instructional tasks and sequences, aligning instruction with the objectives and standards to which the teachers are held accountable). . . .

The process of documentational genesis therefore foregrounds interactions of teachers and resources, and highlights how both are transformed in the course of these interactions.

In ESUM, as well as resources set out above, the scheme of utilization denotes practice in which resources are used, activity takes place, reflection and critical review lead to new elements of innovation, and members of the team learn though participation, reification (objectifying elements of practice; Wenger, 1998) and critical alignment (looking critically at how principles of innovation fit with institutional practice). These together form the documentational genesis in ESUM.

It seems therefore, that theory of community documentational genesis both fits well with the activity in ESUM and offers ways to make sense of the interplay between creating learning opportunities for students and the concomitant development of knowledge and understanding of the teacher in doing so. Research and development go hand in hand both to chart progress and stimulate knowledge in practice. In Figure 1, we see, on the left, a focus on inquiry-based tasks, their use with students and the teacher’s reflection on their use—a cyclic process in which feedback from reflection leads to modification of the tasks to suit students learning. On the right, research analyses the process in its different stages, and also contributes to development.

In this analysis of ESUM in relation to inquiry-based tasks, we recognize the three layers of inquiry designated in previous research into developing mathematics teaching at school level (Jaworski, 2006).

(A) Inquiry in learning mathematics: students engaging with inquiry-based tasks in mathematics to encourage conceptual engagement, learning and understanding.

<table>
<thead>
<tr>
<th>Developmental activity</th>
<th>Developmental research</th>
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<tr>
<td>• Design inquiry-based tasks in which students will engage with mathematics (B)</td>
<td>• What is involved in such design? How does design meet teaching goals? (Data &amp; Analysis) (C)</td>
</tr>
<tr>
<td>• Use tasks with students in pedagogically sensitive ways (A)</td>
<td>• Observe student engagement and pedagogic process (Data &amp; Analysis) (C)</td>
</tr>
<tr>
<td>• Reflect on use of tasks, student engagement and learning outcomes (B)</td>
<td>• Analyse data to show in what ways innovation has met the goals of activity (Analysis) (C)</td>
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Fig. 1. The developmental research process.
(B) Inquiry in teaching mathematics: teachers using inquiry in the design and implementation of tasks, problems and mathematical activity with students, possibly in a project with other teachers.

(C) Inquiry in developing the teaching of mathematics: teachers, researching the processes of using inquiry in mathematics and in the teaching and learning of mathematics—possibly together with outsider researchers.

These inter-related layers provide a representation of developmental activity linking student development through inquiry in mathematics and teacher/teaching development through inquiry in mathematics teaching. We see here a close relationship with the theory of documentational genesis.

4. Innovation in practice

At the start of the semester, students were organized by the lecturer into small groups according to the programmes on which they were registered to facilitate meeting outside of timetabled sessions. They kept to the same groups throughout the module. In tutorials in the first 2 weeks, students met their fellow group members, worked on exploratory tasks employing inquiry-based questions and started to use GeoGebra. These activities, designed to promote the understanding of concepts, introduced elements of the overall style of the module. For example, in their first topic, polynomial functions, students were encouraged to create a range of polynomial graphs and identify characteristics of specific functions and families of functions. Figure 2 shows an example of a task including inquiry-based questions and suggesting use of GeoGebra in tackling the questions. The first question in the task was offered in a lecture to encourage student engagement with lecture content. The second and third parts, originally also designed for the lecture, were seen in practice to be too demanding for addressing in a lecture and so they were included in a later tutorial allowing for more extensive exploration and discussion. We see here an example of critical inquiry (at level C) resulting in modification of design in practice.

Early in the module, students were introduced to their group project for which they would be given some tutorial time and for which some work would be necessary outside module sessions. This included a set of tasks requiring mathematical exploration involving functions. It was to be handed in for assessment towards the end of the module. Figure 3 show one task (of 4) in one variant (of 3) in the project.

Figures 2 and 3 show examples of tasks and questions of an inquiry nature, which were prepared in advance and offered to students in lecture or tutorial according to what seemed most valuable at different times. In tutorials, students worked on such questions in a group and were encouraged to

<table>
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| 1a) Consider the function \( f(x) = x^2 + 2x \) (x is real)  
Give an equation of a line that intersects the graph of this function  
(i) Twice (ii) Once (iii) Never  (Adapted from Pilzer et al. 2003, 7) |
| 1b) If we have the function \( f(x) = ax^2 + bx + c \).  
What can you say about lines which intersect this function twice? |
| 1c) Write down equations for three straight lines and draw them in GeoGebra  
Find a (quadratic) function such that the graph of the function cuts one of your lines twice, one of them only once, and the third not at all and show the result in GeoGebra.  
Repeat for three different lines (what does it mean to be different?) |

Fig. 2. Example of an inquiry-based task using GeoGebra.
communicate with each other, with the lecturer and a graduate student assistant, thus developing a small community of inquiry. Lectures were more formal, delivered by the lecturer using PowerPoint and OHP (for examples), with overt effort to engage students with the lecture material using a range of questions with exercises at key points. Where appropriate, concepts were illustrated using GeoGebra.

Questions during a lecture, asked spontaneously by the lecturer, have been of a rather more direct or closed nature (focusing on immediate concepts) than those pre-designed of a more open or investigative form. For example, in a lecture dealing with rational equations one question was ‘What would I do to get one fraction?’ and in solving quadratic equations, the lecturer asked, ‘Anyone know factors of \(x^2-2x+1\)?’ A few students routinely offer a response. Sometimes this was accepted by the lecturer who then moved on with the lecture. Sometimes the lecturer offered further questions to draw more students into responding and, ideally, to generate further responses from students.

The following account is taken from the lecturer’s reflection on 1 week’s teaching:

In the first example [in the lecture] on Tuesday, I asked students to draw a triangle of given dimensions before going on to consider use of sine or cosine rules. In fact two triangles were possible for the given dimensions. This turned out to be a very good question, since different students wanted to approach it in different ways and we achieved a discussion across the lecture with students in different parts of the room arguing their approach. This seems worth analyzing to reveal the characteristics of a question which achieved this involvement (especially on a Tuesday when students seem more sluggish).

I have included the last sentence above, since it points to one aspect of the wider environment that has to be taken into account in the analysis. We should analyse not only the questions but also the wide range of factors that influence their impact.

3) Now consider this set of fictitious data describing the oscillation of a string (for example a guitar string). [A set of fictitious data follows here.]

i. On a new Geogebra worksheet:
   a. Plot the points
   b. Decide on a function that, in your group’s opinion, provides the best fit curve for the data (e.g., \(x^2\), \(e^x\), \(\sin(x)\), \(\cos(x)\), \(\tan(x)\), \(mx+c\), etc.)
   c. Explore by varying key parameters to find a “best fit” of the function to your data.

ii. Explain what features of the data made you decide on your function

iii. Show the steps taken in order to change the basic function (i.e. \(\exp(x), \sin(x), \cos(x), \tan(x), x^2\), etc.) into your curve (e.g. If you have a line of best fit \(y=5x+16\) then show the graphs of \(f(x)=x, f(x)=5x\) and \(f(x)=5x+16\))

iv. Explain what happens to the curve at each step (How does it change from one step to the next? Does it move up? Does it move across? Does it get bigger? and so on.)

v. Indicate what other families of functions you thought about and explain why they would also work, or would not work, for this data set

vi. How did GeoGebra help you realise what families of functions would work or not work for this data set?

vii. Give a practical example from the field of Materials Engineering where you might see your function in use. You might consult your personal tutor about this.
5. Analysis of data, issues and findings

Analysis to date has addressed students’ responses to two questionnaires (completed by 44 and 40 students, respectively), the lecturer’s weekly reflections on project activity and analysis of one–one and focus group interviews with students. Data from observations of lectures and tutorials are extensive and are being used as a back up to explore further the issues that are arising from the analysis. We discuss below our findings from analysis and related issues.

Most students have found the pace of lectures to be suitable for their understanding and progress, although six suggest inconsistency—they claim that the pace is too slow on topics they had encountered in detail before and too fast on unfamiliar material. More than half report a good level of understanding (38 students found problems in lectures either easy or just right). For some topics, students reported needing extra work in order to understand fully. Only two students reported a very low level of understanding. One computer-based test (on functions), undertaken after relevant topics had been completed, revealed only eight students achieving <60%. A further test on complex numbers and matrices showed a wider spread of results with indications that students found this material more challenging. It seems relevant that most students had knowledge of functions from their A level studies, but had not studied these further topics before.

Analysis of the lecturer’s reflection on ongoing practice reveals a developing awareness (documentational genesis) of modes of innovation and related issues. Findings on the use of GeoGebra show both positive and negative indications. For example, going into GeoGebra mode in a lecture (from PowerPoint) resulted in student attention. Students visibly attended—they stopped writing, talking or shuffling around. However, students indicated in interviews that they found dynamic access to GeoGebra in lectures not to be a good use of time. Some feedback from the relevant staff–student committee had suggested too much use of GeoGebra in the module, so, in the second questionnaire, students were asked if GeoGebra increased their understanding of topics that had been covered. In all, 24 said yes, 4 sometimes, 11 no. Comments included: ‘Makes it easier to see how functions relate to the graphs’, ‘Graphical solutions are overlooked when graphs have to be drawn by hand’, ‘Shows what functions look like - helps understanding’, ‘No, I feel it does not help explain why a given equation gives the curve it does’. Project scripts showed evidence that students saw the use of GeoGebra to be positive in developing their understanding of functions.

In tutorials, some students used GeoGebra rather superficially—for example, they drew many graphs on one set of axes without obvious critical awareness of what they represented or how they were related. Others were able to explain to the tutor what they saw in a graph and were able to relate it to algebraic representations. Observation of small groups revealed that some engaged with the tutorial tasks, and with GeoGebra, in a conceptual way: for an example, see Figure 4, an extract from observation notes. So, there is some evidence of conceptual understanding but it has proved rather hard to quantify or substantiate.

Regarding the use of questioning in the project, students could see the value of different kinds of questions (open, close, inquiry-based) and no strong views were expressed regarding preference. From observations and lecturer reflections, it is clear that considerable spontaneous closed questioning was evident alongside the pre-prepared inquiry-based questions. Together these alternative forms of questions did seem to engage students, and comments from the lecturer, the graduate assistant and the lecturer teaching the module in its second semester suggested that this year’s cohort were more responsive and engaged more evidently with mathematics than previous cohorts.

From the interviews, students indicated that too much time was given to the teaching of functions and too little to the other topic areas which were new to them. While they appreciated some improved understanding of functions, they would prefer more time to have been spent on complex numbers and
matrices. In interviews, students have indicated a strategic approach to their mathematics course; it is important to be able to do what is required to be successful in continuous assessment and to pass the exam. Some believed an instrumental, rather than a conceptual understanding, to be adequate and that when they become engineering practitioners, there would be others to offer mathematical expertise. Some students would have liked more evidence of how mathematics is related to their engineering studies. However, the project, which did pose a question encouraging students to make links, was not universally liked, and the particular question was very poorly addressed in general. Some students felt that they gained enough experience of project work in their engineering modules and it was unnecessary to have a project in mathematics. There was a significant improvement in the examination results for questions based on this module with pass rates for questions increasing from 63% to 93%.

The above findings, issues and comments are representative of the feedback we have gained from students from observations, questionnaire and interviews regarding the innovation studied in ESUM. From comments on the use of GeoGebra and the group project, we shall make modifications to the ways in which these are implemented in the coming year. We shall continue to explore the use of different forms of questioning to promote student engagement and, particularly, to seek to support conceptual understanding. We shall re-order our presentation of content to tackle new topics at the beginning of the module so that students do not become complacent through familiarity with material.

6. Returning to theory

We have tried, above, to make links between the innovatory practices of the project, and the theory we have proposed underpins the project. Here we try to make these links more explicit. The institutional elements of our community of practice include the curriculum and assessment of the module, time allowed and timetabled sessions, locations of lectures and tutorials, student numbers, student culture and expectations, teaching culture and expectations. We are aligned with all of these to some extent and there are differing degrees to which change is possible. For example, curriculum, assessment and time allowed can be changed in the longer term, but sound reasons have to be provided for such change. Findings so far have not produced a compelling case for such changes.

In the ESUM project, we have focused more locally on changing teaching culture and expectations, and to some extent student expectations. Thus, critical alignment can be seen in bringing more interactivity to lectures, introducing inquiry-based tasks and group work, using a computer environment and persuading students of the value of all of these for their learning. We have had some success in promoting a more obvious engagement in terms of response and participation in lectures, which we attribute to styles of questioning and the demands of the group project. The inquiry nature of questions

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\begin{align*}
\text{Task: Draw on a new sheet } & e^{x+1}, \; e^{x+2}, \; 2e^{x+1}, \; e^{2x+3}, \; \ln(x+1), \; 2\ln(3x-1) \\
& \text{The group was working on the above task and noticed that they had entered the } \\
& \text{ln functions incorrectly, omitting brackets e.g. } \ln3x-1 \text{ instead of } \ln(3x-1) \\
& \text{They corrected all the equations they had entered which provoked a discussion } \\
& \text{around the inverse and whether it was symmetric or not. One student did the inverse } \\
& \text{calculation } \\
& y = e^{x+1} \quad \ln y = x+1 \quad x = \ln y - 1 \\
& \text{They plotted this function } (y = \ln x - 1) \text{ in GeoGebra and saw the symmetry }
\end{align*}
\]

Fig. 4. An example of GeoGebra use for conceptual understanding in a small group.
and tasks has sought to engage students more conceptually, but evidence for the success of this is more elusive. Inquiry into teaching approaches has brought teachers towards a more inquiry way of teaching overall, which can also influence students. Moreover, we see critical alignment within the innovation itself as we examine critically the outcomes of innovation and feedback to day to day practice. As our scheme of utilization develops through application of design and critical appreciation of everyday practice, documentational genesis is the transformation that results. We believe there has been transformation this year, perhaps not to the extent we should have hoped, but the resulting knowledge that comes from both practical experience and research findings means that we start another year from a significantly stronger epistemological position.

REFERENCES


Barbara Jaworski is Professor of Mathematics Education in the Mathematics Education Centre at Loughborough University. Her research in mainly into the teaching of mathematics from all levels of school and university education, and in particular into the ways in which mathematics teaching develops. She is especially interested in collaborations between researchers and practitioners in promoting teaching for the effective learning of mathematics. She is a former editor of the Journal of Mathematics Teacher Education and former president of the European Society for Research in Mathematics Education.

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